EEN 366
MODERN
CONTROL

FALL 2007
TEST 2
50 MINUTES

NDU
ECCE DEPARTMENT
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100/100

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NOTE1: OPEN BOOK, OPEN NOTES, CLOSED OLD TESTS AND SOLUTIONS
NOTE2: SHOW ALL WORK IN ORDER TO RECEIVE FULL CREDIT

1. 20 Pts. Use the Routh-Hurwitz Criterion to find the number of roots in the right-half plane of the following Characteristic equation

$$s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4 = 0$$

① 20
② 20
③ 60

s^6	1	-2	-7	-4
s^5	1	-3	-4	0
s^4	1	-3	-4	0
s^3	0	0	0	0

$A(s) = s^4 - 3s^2 - 4 = 0$

$\frac{d}{ds}A(s) = 4s^3 - 6s$

s^4	1	-3	-4	0
s^3	4	-6	0	0
s^2	-1.5	-4	0	0
s^1	-16.667	0	0	0
s^0	-4	0	0	0

1 change of sign ⇒ 1 pole in the RHP ⇒ unstable

2. 20 Pts. A third –order printwheel control system has the characteristic equation

$$s^3 + 1040s^2 + 48{,}500s + 400{,}000K = 0$$

Find the critical values of K for stability . evaluate the roots at these values.

s^3 1 48 500

s^2 1040 400 000 K

s^1 $\frac{50440000 - 400000K}{1040}$ 0

s^0 400 000 K 0

$50\,440\,000 - 400\,000K > 0$

$K < 126.1$

$K > 0$

$0 < K < 126.1$

K must be less then 0

* $s^3 + 1040s^2 + 48500s = 0$ $(K=0)$

$= -48.93 \; j \; s_1 = 0 \; j \; s_2 = -991.062$

* $s^3 + 1040s^2 + 48500s + 50440000 = 0$

$(K = 126.1)$

$$\begin{cases} s_1 = -1040 \\ s_2 = j\,220.22 \\ s_3 = -j\,220.22 \end{cases}$$

3. 60 Pts. Plot the magnitude and phase using Bode- plot techniques for the following transfer function. Label the gain at all break frequencies.

$$G(s)H(s) = \frac{10^{10}\, s^2 (s+5)(s+200)(s+5000)}{(s+1)(s+10)^2(s+500)(s+4000)(s+10000)^2}$$

Find the gain and phase margins and 0-dB crossing(s).

(0.1 ; -32)
(x ; 0)

$32 = 40[\log x - \log 0.1]$

$x = 0.63$

(10^4 ; 40)
(x ; 0)

$-40 = -40[\log x - \log 10^4]$

$x = 10^5$

phase margins:

$s = 0.63$ phase margin $\simeq 180° - 157.5°$
$\simeq 22.5°$

$s = 10^5$ phase margin $= 180° - 180°$
$= 0°$